601. Multiply out using the binomial expansion. The cubic terms should cancel, leaving a quadratic. Test the discriminant Δ.

602. The possibility space is a 6×6 grid of equally likely outcomes. The set of outcomes which produce any particular sum lie along a diagonal.

603. Multiply both sides by $\sqrt{x}+1$, and simplify.

604. Substitute $x = t$ and $y = 1 + \frac{1}{2}t$ into the equation of the circle and solve. This will give you t_1 and t_2, the t values at the intersections. You don't need to work out the (x, y) coordinates. Show that the width of the interval (t_1, t_2) is $^4/_{15}$ the width of the interval $[-2, 1]$.

605. Use the index law $\left(a^b\right)^c \equiv \left(a^c\right)^b$.

606. These quadratics are a pair of *linear* simultaneous equations in the variables x^2 and y^2.

607. Set up NII for each object, and then solve a pair of simultaneous equations.

608. (a) Set up and solve $1 - x \geq 0$,
   (b) Set up and solve $1 - x^2 \geq 0$,
   (c) Set up and solve $1 - x^3 \geq 0$.

609. On the LHS, use

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta},$$
$$\cot\theta \equiv \frac{\cos\theta}{\sin\theta}.$$

Put the fractions over a common denominator and use the first Pythagorean trig identity. $\operatorname{cosec} x$ and $\sec x$ are the reciprocals of $\sin x$ and $\cos x$.

610. Sketch the boundary lines on a graph. The fact that the inequalities are exclusive does not affect the area.

611. Substitute in the values $t = 0, \frac{1}{2}\pi, \pi, \frac{3}{2}\pi$. This will generate the coordinates of the relevant points.

612. Use the formula for the sum of the first n integers, which is
$$S_n = \tfrac{1}{2}n(n+1).$$

613. Consider the fact that the equation $x^2 + 1 = 0$ has no real roots.

614. Find the angle between the two directions of travel. Then use the cosine rule on a velocity triangle to find the speed v at which the ships are separating.

615. Solve the inequality $x^2 - x \leq 0$.

616. Almost any pair of APs will do. Pick e.g. the odd and even numbers. Show that 1×2, 3×4, 5×6 do not have a common difference.

617. A fraction in its lowest terms is zero if and only if its numerator is zero.

618. Consider the greatest and least possible values of $\mathbb{P}(A \cap B')$. You might find a Venn diagram helpful.

619. Find a root first, either by inspection or with a calculator. Then use the factor theorem.

620. Evaluate the definite integrals as normal, using the fact that the integral of $\mathrm{f}(x)$ is $\mathrm{F}(x)$.

621. (a) Subtract the area of the unshaded triangles from the area of the rectangular grid.
   (b) The vertices on the left and right must form a diameter.

622. Consider moments. Resultant force refers only to *translational* equilibrium, as opposed to *rotational* equilibrium.

623. A fixed point of a function f is a value that is fixed i.e. unchanged by the function. So, if α is a fixed point of f, then $\mathrm{f}(\alpha) = \alpha$.

624. Multiply top and bottom of the large fraction by the denominator of the small fractions.

625. Split the fraction up before integrating.

626. (a) The inputs to the square root function must be strictly positive.
   (b) Find the maximum input to the square root.
   (c) The range is the set of possible outputs.

627. (a) The centre is equidistant from $(0, 0)$ and $(4, 0)$, and so must lie on their perpendicular bisector.
   (b) Use Pythagoras to find the squared distance between $(2, y)$ and $(4, 0)$ (first expression for r^2) and the squared distance between $(2, y)$ and $(-2, 6)$ (second expression for r^2).
   (c) Equate the two expressions for r^2, and solve for the radius.

628. The factor is $(x + 1)$.

629. The second derivative is constant in a quadratic, so you know that $\mathrm{g}''(x) = -2$ for all x. Integrate this twice, finding the constant of integration each time.

630. P' is the complement of P, i.e. not-P; $P \setminus Q$ is P minus Q, i.e. P with the elements of Q removed.

631. You might visualise the intervals on a number line. The square brackets mean that the boundaries -1 and 0 are included in the first interval.

632. You are looking for $s = ut + \frac{1}{2}at^2$.

633. Because the second term contains a factor of 2^{-x}, multiply the whole equation by 2^x.

634. These are parallel straight lines.

635. Write the first sentence algebraically, integrate, and substitute into the expression given.

636. Sketch the graphs $y = x$ and $y = \mathrm{g}(x)$.

637. Solve a quadratic equation.

638. Draw in the line $y = x$. Find the coordinates of the three intersections with it. Then use $A_\triangle = \frac{1}{2}bh$ on each of the triangles, with the height running along $y = x$.

639. Percentage error is given by

$$E = \frac{y_{\text{estimated}} - y_{\text{actual}}}{y_{\text{actual}}}.$$

640. Find the length of the diagonals.

641. Express the facts algebraically, then combine them as per solving simultaneous equations.

642. Start with the RHS.

643. Start with $(x+1)^2$, so as to match the quadratic term. Then find k such that $(x+1)^2 + k(x+1)$ gives the right term in x. Continue likewise for the constant term.

——— Alternative Method ———

Let $y = x + 1$. Rearrange to $x = y - 1$ and then substitute for x.

644. Since $x = -a$ is already a root, the right-hand bracket must either have no real roots, or it must be equal to $(x+a)^2$. In the first case, use the discriminant. In the second case, find a, then b.

645. These are both GPs.

646. Weighing scales measure downwards reaction force on them. This force is the NIII pair of the upwards reaction force on the object being weighed. Draw a force diagram for an object of weight mg N.

647. Use the factor theorem.

648. Sketch four cakes, with cuts drawn on them.

649. Find the equation of L, in the form $2x + 3y = k$. Without rearranging, substitute $(6, -6)$.

650. (a) Find a curve with a point of inflection that is not stationary. In other words, find a curve with a point at which the gradient is non-zero and the curvature changes sign.
    (b) Find a curve with a point of inflection that is stationary. In other words, find a curve with a point at which the gradient is zero and the curvature changes sign.

651. Each locus is a straight line. Show that they both pass through the point (b, c), and that they are perpendicular.

652. The answer is yes and no, depending on whether you consider forces *internal* to an object, or if you only consider forces *external* to an object.

653. (a) Set $t = 0$.
    (b) Set $\frac{d}{dt}(k) = 0$.
    (c) Integrate $\frac{d}{dt}(k)$ between $t = 0$ and $t = 60$.
    (d) You need no knowledge of chemistry here: the differential operator $\frac{d}{dt}$ has units of s^{-1}.

654. Use the following setup: if $\mathrm{F}'(x) = \mathrm{f}(x)$, then

$$\int_a^b \mathrm{f}(x)\, dx = \mathrm{F}(b) - \mathrm{F}(a).$$

Show explicitly the effect of switching the limits.

655. A hexagon consists of six equilateral triangles.

656. Use the following formulae:

$$\bar{x} = \frac{\sum x}{n},$$
$$s_x = \sqrt{\frac{\sum x^2 - n\bar{x}^2}{n}}.$$

657. Rearrange to make $\sqrt{2}$ the subject.

658. Consider the graph as a reflection of the standard curve $y = \sqrt{x}$, which is half a parabola.

659. Consider the concept of a limit. The first two are finite changes in t, while the third makes reference to a limiting process.

660. $\mathbb{Q}$ is the rationals.

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661. Sketch the rectangle, shading all points within 1 cm of a vertex. The shaded regions combine to form one circle.

662. Sketch graphs of $y = \text{LHS}$ and $y = \text{RHS}$.

663. Factorise the RHS.

664. Use the inclusion-exclusion formula

$$\mathbb{P}(X \cup Y) = \mathbb{P}(X) + \mathbb{P}(Y) - \mathbb{P}(X \cap Y).$$

Add the fractions over a common denominator to show that, in this case,

$$\mathbb{P}(X) \times \mathbb{P}(Y) = \mathbb{P}(X \cap Y).$$

665. Consider, in each case, the set of values that are to serve as inputs to the function g.

666. Integrate twice with respect to t. Remember to introduce and deal with a constant of integration each time.

667. Consider the restriction to the possibility space, which was originally $\{1, 2, 3, 4, 5, 6\}$, enacted by the piece of information given.

668. (a) Use $y - y_1 = m(x - x_1)$.
   (b) Consider the number of intersections.
   (c) Set $\Delta = 0$.

669. 0! is defined to be 1.

670. Consider the fact that a fraction in its lowest terms is zero iff its numerator is zero.

671. The quartiles are defined by

$$\mathbb{P}(X < X_{\text{lower}}) = 0.25,$$
$$\mathbb{P}(X < X_{\text{upper}}) = 0.75.$$

672. Consider the range of $x \mapsto (x-1)^4$. Alternatively, consider the x coordinate of the turning point.

673. (a) Evaluate the definite integral of x^{-2}.
   (b) The infinite integral is defined as

$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{N\to\infty} \int_1^N \frac{1}{x^2}.$$

   Use part (a) to evaluate this.

674. Set up an equation expressing algebraically the fact that "the ratio between terms 1 and 2 and terms 2 and 3 is the same". You don't need to (and so shouldn't!) call the common ratio r.

675. This is to some extent a trick question. The fact that the car is turning a the corner is irrelevant: there is a consistent relationship, by mathematical definition, between reaction forces and frictional forces.

676. (a) The rate of change of y with respect to x is expressed algebraically as $\frac{dy}{dx}$.
   (b) Substitute into your answer to (a).
   (c) Integrate your equation from (a), using the value of k from part (b). Include a constant of integration.
   (d) Sub in $(0, 6)$ to find the constant of integration. Then sub in $x = 3$.

677. (a) Consider the assumed size and acceleration of the projectile.
   (b) Resolve as $u\cos\theta$ and $u\sin\theta$.
   (c) Set the vertical displacement to zero.
   (d) Use the value for t from part (c) in a horizontal *suvat*.

678. Use the discriminant, or a graphical argument.

679. Solve for intersections, and consider double roots.

680. The set of points equidistant from a pair of lines consists of their angle bisectors.

681. Consider the intersection as an overlap. The key fact is that, on both sides of the formula, the terms count this intersection twice.

682. Use the first Pythagorean identity.

683. (a) Use the formula

$$S_{xx} = \sum x^2 - \tfrac{1}{n}\big(\sum x\big)^2.$$

   Each term can be found using your calculator.
   (b) You can get the variance s^2 directly from your calculator.

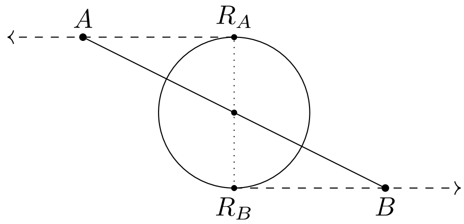
684. Sketch the graph $y = \mathrm{f}(x)$. Of particular concern is whether the domain contains the vertex of the quadratic.

685. The vertices of a quadrilateral should be named in order around the perimeter. Show that, using the student's nomenclature, this is not the case.

686. As it stands, you cannot take the limit, since both numerator and denominator tend to zero. Reduce the fraction to its lowest terms before taking the limit.

"Taking the limit" means sending the value k all the way to 2, which you can't do at the beginning without breaking things.

687. When one lets go, the motion is symmetrical for both. From time $t = 0$, each astronaut travels at constant velocity. The diagram is



688. Consider integration.

689. This is a quadratic in $\sin 18°$. So, solve it as if you were solving a quadratic equation with $\sin 18°$ as the variable. You might find it useful to replace $\sin 18°$ with s.

690. Add the relevant quantities to increase the value of x by 2 and the value of y by 3.

691. Consider the average side length.

692. Set up an inequality and solve. Since the question asks for a set, you should answer in set notation.

693. The function is ill defined when $x^2 + px + q \leq 0$.

694. Consider the line $x + y = k$.

695. Sketch the graphs carefully. Particularly, whether there are intersections or not will depend on the fact that the gradient of $x - 2y = 0$, which is $\frac{1}{2}$, is shallower than the gradients ± 1 of the mod graphs. Make sure your sketch shows this.

696. Use the arithmetic mean information to set up an equation in p and q. Find p in terms of q. Then use this to simplify expressions for the difference and for the geometric mean.

697. Express the translation with a vector.

698. (a) Set up a pair of simultaneous equations, noting that the accelerations are different.
 (b) Consider the different accelerations.

699. This is an infinite GP.

700. (a) Put the equation as $(x - a)^2 + (y - b)^2 = r^2$, which is a circle radius r centred on (a, b).
 (b) There is no need to use *coordinate* geometry here. You can convert directly from the radius to the lengths of the diagonals of the square, then to the side length, then to the area.

——— End of 7th Hundred ———